COMPUTATIONAL STUDY OF BOUNDARY LAYER FLOW OF NANOFLUIDS OVER STRETCHING SURFACES

SYNOPSIS

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Rekha Devi
Enrollment No.: H14123

Under the supervision of

Dr. Shilpa Sood
Associate Professor

DEPARTMENT OF MATHEMATICS & STATISTICS
SCHOOL OF BASIC AND APPLIED SCIENCES
CAREER POINT UNIVERSITY HAMIRPUR
TIKKER (KHARWARIAN), BHORANJ
HAMIRPUR (H.P.)-176041, INDIA

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1 Introduction

Fluid mechanics is a broad area of Mathematics, which examines the physical characteristics of fluids as well as their motions in various physical situations.The study of motion and equilibrium of fluids, specifically of liquids and gases, is covered in the field of fluid mechanics, which extends the concepts gained in mechanics and thermodynamics. Since Euler published his famous equations of fluid flow for ideal fluids in his paper "General Principles of the Motion of Fluids" in 1755, this topic has seen tremendous advancement, changing the wide range of engineering applications. In the fields of science, engineering, and technology, fluid mechanics has attracted significant attention. For many domains of science and engineering, including meteorology, oceanology, aerodynamics, biofluids, propulsion and combustion, particles physics, and acoustics, fluid mechanics established the groundwork. Due to its numerous applications in mathematics, physics, engineering, and other fields, fluid dynamics is thought of as a progressing topic of study.

Numerous industrial processes might benefit from the research of various fluid types, boundary conditions, and the effects of characteristics like viscosity, porosity, magnetic field, etc. on the flow velocity and temperature fluctuations in the fluid medium (Cohen & Kundu, 2004). Designing any sort of fluid machinery, including turbines, fans, pumps, compressors, lubrication systems, shopping centres, aircraft for subsonic and supersonic flights, submarines, etc., requires knowledge of fluid mechanics. Numerous fields, including the aerospace industry, power generation, nanotechnology, wind energy, and wind farms, among others, use fluid dynamics.Also fluid dynamics has vast range of applications such as in fluid mixtures and heat ex-changers, flying robots, aerospace industry, power generation, nanotechnology, wind energy and wind farms, etc (Durst, 2008). Therefore, the topic has recently caught the interest of b oth scientists and engineers. Ludwig Prandtl, a German scientist, is credited with developing the theory of laminar boundary layer flow. He postulated that for fluids with very low viscosities, the flow past a solid body can be divided into two regions: (i) a very thin layer immediately surrounding the body (known as the boundary layer), in which the viscous effects are considered significant; and (ii) the region beyond this layer, in which the viscous effects may be regarded as negligible.The boundary layer equations, which Prandtl simplified into a theoretically manageable form, are the Navier-Stokes equations. Boundary layer theory is very helpful to calculate pressure drag of the object, which is created due to the change in pressure distributions because of flow separation. This theory is also helpful to determine the shape of the object in order to prevent flow separation. The Navier-Stokes equations are used to simulate low-speed, high-speed, compressible or incompressible, inviscid or

viscous flows. They are based on the idea of momentum conservation (Prandtl, 1904). The governing equations for the unsteady flow of viscous compressible fluids are:

• Equation of State

$$p = \rho RT,$$

• Equation of Continuity

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j}(\rho v_j) = 0,$$

• Equation of Motion (Navier-Stokes Equations)

$$\rho \frac{\mathbf{D}}{\mathbf{Dt}}(v_i) = \rho f_i - \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\mu\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} - \frac{2}{3}\frac{\partial v_k}{\partial x_k}\delta_{ij}\right)\right],$$

• Equation of Energy

$$\rho \frac{\mathbf{D}}{\mathbf{Dt}}(C_p T) = \frac{\mathbf{D}p}{\mathbf{Dt}} + \frac{\partial Q}{\partial t} + \frac{\partial}{\partial x_i}\left(\kappa \frac{\partial T}{\partial x_i}\right) + \phi_1,$$

Here, $\dfrac{\mathbf{D}}{\mathbf{Dt}} = \dfrac{\partial}{\partial t} + v_j \dfrac{\partial}{\partial x_j}$ is material derivative, $i, j = 1, 2, 3$ and

$$\phi_1 = \mu\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} - \frac{2}{3}\frac{\partial v_k}{\partial x_k}\delta_{(ij)}\right)\frac{\partial v_i}{\partial x_j}$$

. Here v_i are the velocity components and x_i are the spatial components in x, y, z-directions respectively, t is time, f_i are the body forces, κ is the thermal conductivity, C_p is the specific heat at constant pressure, T is temperature, μ is the dynamic viscosity, ρ is the density, p is the pressure, δ_{ij} is Kronecker delta, Q is the heat added per unit mass of fluid through conduction, R is characteristic gas constant. Above equations for the unsteady flow of viscous incompressible fluids take the following form:

$$\frac{\partial v_j}{\partial x_i} = 0,$$

$$\rho \frac{\mathbf{D}}{\mathbf{Dt}}(v_i) = \rho f_i - \frac{\partial p}{\partial x_i} + \mu \frac{\partial^2 v_i}{\partial x_j \partial x_j},$$

$$\rho C_p \frac{\mathbf{D}}{\mathbf{Dt}}(T) = \frac{\partial Q}{\partial t} + \kappa \frac{\partial}{\partial x_i}\left(\frac{\partial T}{\partial x_i}\right) + \phi_1,$$

where

$$\phi_1 = \mu\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right)\frac{\partial v_i}{\partial x_j},$$

and $i, j = 1, 2, 3$. Depending on their viscosity, fluids can also be divided into Newtonian and non-Newtonian fluids. Shear stress and shear strain have a linear relationship, according to Sir Isaac Newton's (1642–1727) fundamental fluid law (Renn, Damerow & McLaughlin, 2002). Shear stress is directly proportional to the rate of shear strain. It can be mathematically represented as follows:

$$\tau = \mu \frac{du}{dy},$$

where μ is dynamic viscosity and it remains constant at all shear rates at a constant temperature. Newtonian fluids are defined as those that follow this law. Newtonian fluids include substances with a constant viscosity, such as water, oil, alcohol, and gasoline. The viscosity of some fluids, however, defies Newton's equation and varies in reaction to applied shear. The vast majority of the fluids we come into contact with on a daily basis are non-Newtonian fluids, which means that their viscosity varies according to the shear stress or force being applied. These show a nonlinear relationship between shear stress and shear strain and do not follow Newton's equation of viscosity (Anderson, 2005).

1.1 History

There were a number of flow difficulties that early civilizations could easily solve. Ancient irrigation systems are something we are familiar with.Archimedes (285-212 B.C.) created the laws of buoyancy and the parallelogram, which he then applied to both submerged and floating objects. Leonardo da Vinci (1452-1519) proposed mass conservation equations and tested them using jets, hydraulic jumps, waves, etc. for one-dimensional steady flow. Newtonian fluids have equations of motion and viscosity that were developed for linear fluids by Sir Isaac Newton (1642–1727). Many of his works contained accurate descriptions of jets, waves, eddy formation, etc. The Newtonian fluids' laws of motion and viscosity were developed by Isaac Newton. Leonhard developed the differential and integral form of the equation of motion that is today known as the Bernoulli Equation.For anybody immersed in an inviscid potential and an incompressible flow, the drag force is zero, according to a famous paradox created by Alembert using partial differential equations, which he developed mathematically. Osborne Reynolds demonstrated the significance of the dimensionless number, which was given his name as Reynolds' number, and Lord Rayleigh proposed the method of dimensional analysis. The Navier-Stokes equations are a system of non-linear partial differential equations that were developed by altering Euler's equations to integrate the effects of internal friction in a flow field. The first time viscosity was formally introduced was in Stokes' derivation.

But getting these equations' analytical solutions was essentially impossible. When Orville and Wilbur Wright created the first usable aeroplane in 1903, they had to figure out how much lift and drag it would need. Determine the pressure and shear stress distributions, then integrate them over the surface to calculate the force.

1.2 Prandtl's Boundary Layer

In analysing the behaviour of fluids, the boundary layer is a crucial factor. A boundary layer is a thin layer of a moving gas or liquid that comes into contact with a solid surface, such the inside of a pipe or the wing of an aeroplane. The issues with boundary layer flow have been thoroughly investigated over the past few years, and scientists are now actively working to identify solutions that are equivalent for flow past a boundary layer. German scientist Ludwig Prandtl created the boundary layer theory notion in 1904. It wasn't until Ludwig Prandtl's remarkable presentation "On the Motion of a Fluid of Very Small Viscosity" was given in 1904 at Heidelberg's Third International Congress of Mathematicians that the motion of real fluids with small viscosities was truly understood. Prandtl asserts that for fluids with low viscosity, the flow around a solid body can be separated into two areas: (i) A thin layer immediately around the body, known as the boundary layer, where viscosity plays a significant role. (ii) The region outside of this layer has an irrotational flow and is subject to the perfect fluids theory. Re, which measures the ratio of inertia forces to viscous forces, must be high in order for the boundary layer to exist. If the Reynolds number is really high, the flow won't always remain laminar and turbulence could happen. On the other hand, if the Reynolds number is low, there won't be a boundary layer since the viscous forces will be strong and will affect the entire flow field. It should be noted that, in contrast to laminar flows, where fluid motion is orderly, turbulent flows are characterised by the fluid particles showing irregular motion, leading to random variations in both space and time of flow variables such as velocity, pressure, etc. The no-slip requirement, which Prandtl enforced, stipulates that the fluid sticks to the walls and, as a result, the total velocity at the wall is zero or equal to the velocity of the body, at the limits of the solid body. This is caused by the intermolecular attractions present in genuine fluids, which result in stretching tensions. Since the no-slip criterion is enforced at the wall and the velocity outside the boundary layer is that of potential flow, a substantial velocity gradient would actually exist across the thin boundary layer.

Due to the high gradient, the boundary layer experiences increased shearing pressures, and even a tiny viscous force has a significant impact on the boundary layer. Since there is no variation in pressure normal to the wall, the potential flow outside the boundary layer determines the pressure at any given place inside it. The boundary Layer theory idea, which

established a new chapter in the development of fluid mechanics. Applications of Prandtl Prandtl boundary layer theory that are extremely important include the computations of drag forces on flat plates, drag on ships, drag on aeroplanes, etc. The boundary layer theory is highly useful for calculating the object's pressure drag, which is caused by a change in pressure distributions brought on by flow separation. In order to avoid flow separation, the geometry of the item can be determined using boundary layer theory. Numerous parameters, including flow separation, skin friction drag, object pressure drag, etc. on the body over which the fluid is flowing, were calculated thanks to this theory. He investigated the influence of viscosity on laminar flow on a flat plate and demonstrated that for mildly viscous fluid, viscosity plays a significant role in the boundaries and is negligible in the majority of the flow (Prandtl, 1904).

The work of Ludwig Prandtl demonstrated how the Navier-Stokes equations may be streamlined to produce the boundary layer equations. In reality, the coupled, non-linear partial differential equations that made up the boundary layer equations still existed. This is an important idea that significantly reduced the degree of nonlinearity and altered the mathematical characteristics of the Navier-Stokes equations. The principles of momentum conservation, mass conservation, energy conservation, linear momentum, angular momentum, and the second law of thermodynamics provide the foundation of the Navier-Stokes equations. are used to depict flows that are incompressible or compressible, moving at low or high speeds, or that are viscous or inviscid. With the exception of a few special circumstances, it is quite challenging to find the exact solution to these equations because of their highly nonlinear nature. Only in a few circumstances are exact solutions available before removing viscous words. The boundary layer can also be described as a layer in which the fluid particles are moving relative to one another. Another way to describe the outer flow is as a potential flow (Rosenhead, 1963). the development of boundary layer flow with mass transfer over a porous medium over a moving permeable flat plate with varying wall concentration while a first order chemical reaction is occurring. Both the Keller box approach and the method based on the fourth order Runge- Kutta iteration scheme with shooting method are used to numerically solve the nonlinear system of the nonlinear differential governing equation (Keller, 1978). Prandtl's work reduced the Navier-Stokes equations to a simpler form, now known as boundary layer equations. One of Prandtl's pupils, Hartman Blasius, solved these reduced boundary layer equations for laminar flow across a flat plate, leading to equations for skin friction drag and boundary layer thickness (Blasius, 1908). The two-dimensional Blasius flow for a wedge flow was extended by Falkner and Skan. The issue was subsequently expanded for axisymmetric

and three-dimensional flows by numerous eminent academics (Falkner & Skan, 1931). All of these flows, whether they occurred on a moving or static surface, were examined under the presumption that an external potential flow existed. Sakiadis was the first to look at the case where the fluid is static for two-dimensional and axisymmetric flow. The Blasius flow was exactly the contrary in this example. Sakiadis' analysis revealed that in this instance, various physical parameters had distinct values. For instance, viscous drag was significantly higher in this case than it was in the Blasius ow, demonstrating that Blasius solutions cannot be applied in this particular scenario and that they must instead be explored separately. On continuous solid surfaces, boundary layer flow was researched by Sakiadis. According to his observation, this form of flow represents a new class of boundary layer problems, with solutions that are significantly different from those for boundary layer flow on surfaces of finite length (Sakiadis, 1961). The flow and heat transfer characteristics of incompressible viscous flow across a non-linearly extending sheet in the presence of viscous dissipation were quantitatively examined by Alinejad and Samarbakhsh. Additionally, they investigated the impact of different Prandtl number and nonlinear stretching parameter values (Alinejad & Samarbakhsh, 2012).

1.3 Boundary Layer Equations

In fluid mechanics, the law of conservation of mass is very important. This law states that mass can be neither created nor be destroyed in a specific control volume with having no source or sink. The equation of conservation of mass is obtained as For a two dimensional unsteady incompressible fluid flow, the equationsof continuity and mass conservation are given as (Schlichting & Gersten, 2015).

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right), \tag{2}$$

$$z\frac{\partial u}{\partial t}+u\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right), \tag{3}$$

where u and v are the velocity components, x and y are horizontal and vertical coordinates respectively, p is the pressure, t is the time, ρ is the density and ν is the kinematic viscosity. xy-plane is the plane of motion, x coordinate is along the solid boundary and y coordinate is perpendicular to it. Here we will obtain the asymptotic form of the Navier–Stokes equations at very high Reynold's number. Since the boundary surface is non-porous and there applies no-slip condition, the velocity components u and v both

must vanish at $y = 0$. For a two-dimensional unsteady incompressible flow, the Prandtl's boundary layer equations are

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} + \frac{\partial^2 u}{\partial y^2}$$
$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

1.4 Nanofluids and their models

The term "nanofluid" was initially used by Choi & Eastman (1995) to describe a novel technique for improving the thermal conductivity of heat transfer fluid. The suspension of nanoparticles in fundamental fluids is known as nanofluids. The basefluid's thermal characteristics are improved by the inclusion of nanoparticles. Non-metals, single- or multiwall carbon nanotubes, oxidised ceramics, and other nanomaterials are used to make nanofluids, whereas base fluids include water, glycol, oils, and lubricants (Choi & Eastman, 1995). Fast heat transfer occurs in the cylindrical particles because of their greater distance ratio (length to diameter ratio), which improves their thermal conductivity in comparison to spherical particles (Xie et al., 2002). The alumina-water nano fluid's viscous characteristics led researchers to the conclusion that viscosity rises as particle concentration does (Das, Choi& Patel, 2006).The viscosity of copperoxide, ethylene glycol, and water nanoparticles was investigated by Namburu et al. in 2007. As a result, the viscosity of nanofluids increases with the addition of nanoparticles. The viscosity of nanofluids is enhanced by the particle volume concentration, according to Sundar et al. (2013). According to Nguyen et al. (2008), typical fluids have weak cooling capabilities and two particles of different sizes have varying viscosities, which results in limited and controlled heat transfer. Engineers and scientists have put a lot of work into finding a solution to the problem of limited heat transfer. Because solids have better thermal conductivity than liquids, Kakac & Pramuanjaroenkij (2009) was one of the suggested methods to increase heat transfer rates. As a result, Maxwell proposed the notion of suspending microscopic particles in the base fluids, which increased the rate of heat transfer. However, this suspension causes a number of issues, such as the settling of these particles in the carrier liquid, an increase in pressure drop, and an improvement in thermal resistance (Maxwell, 1904).

Masuda et al. (1993) observed an increase in fluid thermal conductivity with the addition of seven submicronic particles, although there were certain drawbacks, such as the submicronic particles' tendency to settle and aggregate in the fluid.By dispersing nanoparticles in the base fluids, Choi (1995) overcame Maxwell's 1904 suspensions' flaw

(Masuda et al.,1993). Choi was the first to coin the term "Nanofluid" to describe this new category of heat transfer fluids. The brownian motion of suspended nanometer-sized particles helps to overcome difficulties like aggregation and settling of particles caused by gravity. Nanofluids are designed fluids that contain solid particles suspended in base fluids that are less than 100 nm in size and whose concentration should be less than0.5 volume percent. The applications for which the nanofluids are being manufactured completely determine the selections of the base fluids and nanoparticles used in the synthesis of the nanofluids. Since nanoparticles are comprised of metal and their oxides, one can use water or any other organic solvent as the base fluid. There is a wealth of research that demonstrates the value of employing nanofluids as a coolant in a variety of applications where heat transmission is necessary. Nanofluid is used as a coolant in a variety of devices, including solar reflectors, nuclear reactors, automobile engines, and microelectronic cooling systems. To analyse the convective heat transmission in nanofluids, the Tiwari and Das and Buongiorno models are. Nanofluids are the suspension of nanoparticles in base fluids. The addition of nanoparticles enhances the thermal properties of the basefluid. The nanomaterials which frequently used (Buongiorno, 2006; Tiwari & Das, 2007). The following provides an overview of these models:

1.4.1 Tiwari and Das nanofluid model

This nanofluid model is conferred by Tiwari and Das and is also termed as either homogeneous nanofluid flow model or single component flow model because nanofluids act like liquids. In this model, Tiwari and Das concluded that improvement in the heat transfer characteristics of nanofluids is basically due to incremented thermal conductivity of fluids (Tiwari & Das, 2007; Das, 2013). Following are the conservation equations for the flow of nanoliquids structured by Tiwari and Das nanofluid model:

- Continuity Equation:

$$\nabla . u = 0, \tag{4}$$

- Momentum Equation:

$$\rho_n \frac{Du}{Dt} = -\nabla P + \mu_n \nabla^2 u, \tag{5}$$

- Energy Equation:

$$(\rho c_p)_n \frac{DT}{Dt} = \nabla \cdot \kappa_n \nabla T. \tag{6}$$

1.4.2 Buongiorno nanofluid model

Buongiorno discussed the seven slip mechanisms that can attributed towards the production of the relative velocities between carrier fluids and nanoparticles. But he concluded that out of these seven slip mechanisms only Brownian motion and thermophoresis can produce relative velocity (Sheremet & Pop, 2014) . Brownian motion is the random movements of the particles in the carrier liquid and the mass flux due to Brownian motion is calculated by

$$\mathbf{j}_{p,B} = -\rho_p D_B \nabla \phi, \tag{7}$$

Thermophoresis is a phenomenon of diffusion of nanoparticles with a change in temperature and the mass flux due to this thermophoretc effect is

$$\mathbf{j}_{p,T} = -\rho_p D_T \, \mathbf{V} = -\rho_p D_T \frac{\nabla \mathbf{T}}{\mathbf{T}}, \tag{8}$$

On combining equation (18) and (19), we get $\mathbf{j}_p$, which is diffusive mass flux of nanoparticles. Mathematically the expression for $\mathbf{j}_p$can be written as:

$$\mathbf{j}_p = \mathbf{j}_{p,B} + \mathbf{j}_{p,T} = -\rho_p D_B \nabla \phi - \rho_p D_T \frac{\nabla T}{T}, \tag{9}$$

2 Literature Review

The study of fluid flow over a stretching sheet has been very popular recently due to its applicability in several industrial fields. Engineering applications for boundary layer flow over a stretching sheet include the production of plastic films, the coating of emulsion on photographic films, and the fibre industry (Yu et al., 2009).The flow across a stretching body is significant in operations such as production, extrusion, condensation, etc. in the plastics and metals industries. A stretched sheet interacts mechanically and thermally with the fluid during the manufacturing process. The heat transmission mechanism and flow have a big impact on the final product (Pal, 2009). Numerous mathematical problems have been solved using analytical or numerical methods.These issues have appeared in fields as diverse as fluid dynamics, mechanical engineering, chemical engineering, theoretical physics, and aeronautical engineering (Vajravelu & Mukhopadhyay, 2015). In a nanofluid with viscous dissipation, Muhmmad et al. (2014) and Ishak et al. (2015) examined the boundary layer flow across a moving plate numerically. it's shown however

the Prandtl variety, plate speed parameter, pedesis parameter, thermophoresis parameter, Eckert variety and Lewis variety have an effect on the worth of reduced Nusselt and Sherwood variety and also the skin friction constant likewise because the concentration and temperature profiles.

Bhattacharyya (2012) have analyzed the results of partial assume steady boundary layer stagnation-point flow of an incompressible fluid and Heat transfer towards a shrinking sheet. The requirements for the non-existence, existence, uniqueness, and duality of the solutions to self-similar equations square measure investigated numerically during this work. because of the rise of slip parameter (d), the range of velocity ratio parameter (c/a) where the similarity solution exists, increases.A mathematical model of the steady boundary layer flow of nanofluid caused by an exponentially permeable stretched sheet with an external magnetic field was presented by Bhattacharyya & Layek (2014). In this model, he took into account how heat transfer and nanoparticle volume friction are affected by Brownian motion and thermophoresis. Khan and Pop (2011) looked statistically into the issue of laminar fluid flow, which is caused when a flat surface stretches in a nanofluid. He used a nanofluid model that takes into account the effects of thermophoresis and Brownian motion. It has been observed that the reduced Nusselt number is a decreasing function of each dimensionless number,while the reduced Sherwood number is an increasing function of higher Pr and a decreasing function of lower Pr number for each Le, Nb and Nt numbers

The steady boundary layer flow of an incompressible viscous flow across a linearly stretched plate was initially studied by Crane (1970). Under varied physical constraints, Gupta& Gupta (1977), Char (1988), and Pavlov (1974) continued Crane's work.Hayat et al. have examined the magneto hydrodynamic boundary layer flow of a second-grade fluid over a contracting sheet (2007). It has been discovered that the governing nonlinear issue is solved for the series solution using the homotopy analysis approach, and the convergence of the resulting solution is explicitly investigated. Wang (2008) investigated the stagnation flow approaching a sheet that was contracting. He gave axisymmetric stagnation flows as well as two-dimensional stagnation flows some thought. It has been noted that solutions don't exist for higher shrinkage rates and might not be unique in the case of two dimensions. The flow structure is complicated by the stagnation flow and the shrinking sheet being out of alignment. Due to an increase in boundary layer thickness, convective heat transmission diminishes as the rate of shrinkage rises. The effect of chemical reaction and convective boundary conditions on the thermosolutal behaviour of magneto hydrodynamic (MHD) fluid flow over a stretching sheet was examined by Matta & Gajjela (2018). Various flow factors, including the magnetic parameter, vortex viscosity

parameter, order of chemical reaction, Prandtl, skin friction coefficient, and Eckert number, were explored in terms of their physical significance.

Ryzhov and Savenkov (1987) looked at how a wave packet from a point source would move through a flat plate's boundary layer.He constructed the field of perturbed motion by using the framework of the linearized theory of the boundary layer with self-induced pressure. The fluid is assumed to be incompressible, and the distance from the leading edge of the plate was chosen to be so great, so that the Reynolds number can be assumed to tend toward infinity.Gbadeyan et al. (2011) explored the boundary layer flow created in a nanofluid by a linearly extended sheet in the presence of thermal radiation and an induced magnetic field after doing a similarity analysis of the transport equations. The transport model utilised takes into account Brownian motion and thermophoresis. The analysis shows that the temperature and concentration profiles in the corresponding boundary layers depend on five additional dimensionless parameters in addition to the Prandtl and Lewis numbers. These parameters are the Brownian motion parameter Nb, the thermophoresis parameter Nt, the thermal radiation parameter Ra, the local magnetic field parameter Mx, and the convection Biot number Bi.

Amin and Das (2013) explored the evolution of an unsteady boundary layer over a flat plate under the impact of wake vortices produced by an elliptic cylinder for various Reynolds numbers. To see the impact of a cylinder in the laminar boundary layer over a flat surface, where the cylinder controls the laminar boundary layer flow, researchers have analysed the interaction between the boundary layer generated over a flat plate and the cylinder wake. The computations led to the discovery of two distinct boundary layer-wake interactions: a strong wake-boundary layer interaction and a weak wake-boundary layer interaction. The Reynolds number and plate stream-wise placement have a significant impact on the interaction's intensity. The wake loses coherence more quickly and the boundary layer is more susceptible to perturbation as the Reynolds number increases.The conduction of heat and flow caused by an exponentially stretched sheet was explained by Abd El-Aziz (2009). Additionally, other scholars expanded the issue to include . The case of a porous surface that is exponentially extending was taken into consideration by Elbashbeshy (2001). The effects of thermal radiation on heat transfer and stable boundary layer flow over an exponentially stretched sheet were investigated by Bidin & Nazar (2009).

Timoshin (1997) looked into the non-Newtonian fluid's stability in a boundary layer over a thin sheet of several fluids. He took into account the two-fluid system and used the triple-deck theory to evaluate it. This study focused on the instability that develops

in two-fluid flow systems, and a similar investigation was carried out using thin films of non-Newtonian fluids. By utilising boundary layer analysis, Hassanien et al. (1998) investigated the power-law fluid problem of heat and flow transmission over a stretching sheet with changeable wall temperature. The energy equation and Navier-Stokes equation were first simplified using similarity transformation, and the governing system of the ODE was then solved using the Chebyshev polynomial expansion (non-linear). Through parametric experiments, the effects of the Prandtl number, surface temperature, rate of mass transfer at the surface, and flow index were examined. It was determined that the fluid characteristics affected heat transport and friction factor. The laminar and steady flow of a second-grade liquid over a radially stretched sheet was explored by Ariel (2001). In this study, it was demonstrated that the boundary value problem and boundary conditions both result from the viscoelasticity of fluids and the order of the differential equations, respectively. It was determined that the solution existed for all viscoelastic parameter values. Both asymptotic solutions appropriate for large viscoelastic parameters and perturbation solutions for tiny viscoelastic parameters are also achieved. The brief history of boundary layer theory and numerical method (Keller Box Method) for solving boundary layer equations can be studied through the paper of (Tani,1977) & (Keller, 1978).

Schowalter (1960) & Acrives et al. (1960) was firstly described the power-law fluid with the theory of boundary layer flow. It was concluded that the self-similar solutions exist for the boundary layer equations of power-law fluid flow when the flow is of Falkner-Skan type.Ibrahim (2017); Mahapatra & Gupta (2002) examined the melting heat transfer in Magnetohydrodynamic stagnation point flow of a nanouid past a stretching sheet. Boundary layer theory is employed to simplify the system of partial differential equation of motion, energy and concentration to three coupled non-linear ordinary differential equations. The effect of pertinent parameters on different flow fields are determined and discussed in detail through several plots and tables. It was determined that when the flow is of the Falkner-Skan type, the self-similar solutions exist for the boundary layer equations of power-law fluid flow.A nanouid flowing across a stretching sheet in a magnetohydrodynamic stagnation point flow was studied by Ibrahim (2017) and Mahapatra& Gupta (2002) for melting heat transfer.The system of motion, energy, and concentration partial differential equations is reduced to three coupled non-linear ordinary differential equations using boundary layer theory. Through several charts and tables, the impact of relevant parameters on various flow fields is determined and thoroughly explored.

3 Research Gap Identification

The idea of two-dimensional nanofluid boundary layer flows over stretching sheets will be investigated in this research endeavour. Laminar boundary-layer flow past a stretching sheet has attracted a lot of attention during the past few decades. Due to its numerous industrial applications, flow, heat, and mass transmission over a stretching sheet has drawn a lot of interest. It also has a significant impact on a number of technical and natural processes. Several researchers used the boundary-layer concept to produce results that closely match the experimental findings. Many thermal engineering applications, including geothermal systems, crude oil extraction, ground water contamination, thermal insulation, heat exchangers, the storage of nuclear waste, etc., frequently involve boundary-layer fluxes over stretching. The consideration of boundary-layer issues for two-dimensional steady and incompressible laminar flow past a stretching surfaces is of particular importance in the science of aerodynamics. The focus has been on boundary layer flows of nanofluids for many years, and a lot of work has been done in this area. However, there is a significant gap in the literature that has to be addressed and many themes remain unexplored. Shape and size of the nanoparticles as well as external variables like pressure, temperature, and surface responses are some elements that will open up new avenues for research in this field. The flow heat and mass transport phenomena will be better understood with the use of numerical solutions for boundary layer flows of nanofluids. The project will show how different dimensionless factors and the volume percentage of nanoparticles affect the velocity, temperature, and concentration profiles. The main goal of this study is to present ideas and theories that can be used to precisely control fluid flow properties and accelerate mass and heat transfer rates. Shape and size of nanoparticles, in addition to external variables like pressure, temperature, and surface interactions, are some elements that will open up new avenues for research in this field. The flow heat and mass transport phenomena will be better understood with the use of numerical solutions for boundary layer flows of nanofluids. The main goal of this study is to present ideas and theories that can be used to precisely control fluid flow properties and accelerate mass and heat transfer rates.

4 Objectives

In this study, we will concentrate on comprehending how nanofluid flows over stretched sheets behave in boundary layers. Boundary layer flow past stretching sheets is a crucial topic that requires additional exposure in the future within the context of boundary layer theory. The primary reason chose this field as my study focus is due to its countless

applications in various manufacturing, industrial, automotive, transportation, extrusion, and other industries. In this study, we will concentrate on comprehending how nanofluid flows over stretched sheets behave in boundary layers. The following will be the study's primary goals:

1. To comprehend the effects of physical characteristics on nanofluid flows through exponential stretching, such as magnetic field strength, porous medium permeability, chemical reaction, radiations, and thickness parameters.

2. To illustrate and thoroughly discuss the results that were obtained, as well as to look into the effects of convection, porous medium permeability, and nanofluid characteristics on the flow of nanofluid through stretching sheets.

3. To comprehend how flow past the stretching sheet is affected by controlling factors such convection, magnetic field, and porous medium permeability; and to investigate the implications of relevant physical parameters on flow past the stretching sheet.

5 Proposed Methodology

With the aid of several similarity transformations, the dimensional boundary layer equations derived from the physical problem will be converted into the dimensionless equations. After that, we will solve these non-linear boundary layer equations using an implicit finite difference method. In this research project, we will use the Keller box method and the Bvp4c Solver in Matlab Software to solve these equations (KBM). For the acurate solution of the parabolic differential equations, this method is highly recommended because it has second-order accuracy and unconditional stability.The procedures for utilising KBM to solve differential equations are as follows:

1. Firstly, the governing differential equations will be reduced into a first-order system.

2. Secondly, the difference equations for obtained first-order system will be written with the help of difference approximations.

3 We will derive the governing boundary layer equations , then execute similarity transformation to find the similar solutions.

4. To develop computational models of the relevant issues using the Buongiorno/Tiwari and Das nanofluids model. To utilize the similarity transformations in order to find the self-similar equations of the governing mathematical model.

5. Using Newton's method to linearize the resulting equations and LU decomposition to solve the resulting system. Finally, MATLAB is used to plot the data.

6 Thesis Outline

This thesis is concerned with the computational study of the boundary layer flow of nanofluids over stretching surfaces and will contain six chapters as follows:

Chapter 1 (introduction): The theory of boundary layers, the flow of nanofluids, the flow through porous media, and solution methods will all be covered in-depth. Additionally, it will describe the precise governing equations, study objectives, and research gap that led to the necessity of the current study.

Chapter 2 (Literature Review) :

This chapter will review the literature on boundary layer flow over a stretching sheet in nanofluids, the conduction of heat and flow due to an exponentially stretching sheet, boundary layer theory, and numerical methods for solving boundary layer equations. This chapter will also review the literature on old and latest findings of various researchers in these fields.

Chapter 3 (Flow Past Stretching Sheet) :

In this chapter, we will use Buongiorno nanofluid model for the modeling of nanofluids past a stretching sheet. Hence we will use Bvp4c technique to find the numerical solutions of the stated work.

Chapter 4 (Flow Past Exponential Stretching Sheet):

This chapter will include the flow of nanofluids through a porous surface past a nonlinear stretching surface.

Chapter 5 (Flow Past Osciilatory Surface):

In this chapter we will make the use of Bvp4c technique for solving the nanofluids model past an osciilatory surface.

Chapter 6 (Conclusion):

This chapter will comprise the conclusion of whole study based on objectives along with summary.

References

Abd El-Aziz, M. (2009) Viscous dissipation effect on mixed convection flow of a micropolar fluid over an exponentially stretching sheet. *Canadian Journal of Physics*. 87 (4), 359–368. Available from: https://doi.org/10.1139/P09-047.

Acrivos, A., Shah, M. J. & Petersen, E. E. (1960) Momentum and heat transfer in laminar boundary-layer flows of non-Newtonian fluids past external surfaces. *AIChE Journal*. 6 (2), 312–317. Available from: https://doi.org/10.1002/aic.690060227.

Alinejad, J. & Samarbakhsh, S. (2012) Viscous flow over nonlinearly stretching sheet with effects of viscous dissipation. *Journal of Applied Mathematics*. Available from: https://doi.org/10.1155/2012/587834.

Amin, S. & Das, D. K. (2013) Numerical Investigation of the Wake Affected Unsteadiness on Flat Plate Boundary Layer. *Journal of Mechanical Engineering*. 43 (1), 15–22. Available from: https://doi.org/10.3329/jme.v43i1.15771.

Anderson, J. D. (2005) Ludwig Prandtl's boundary layer. *Physics Today*. 58 (12), 42–48. Available from: http://jontalle.web.engr.illinois.edu/uploads/493/PDFs/PrandtlBoundLayer.05.pdf. [Accessed 20th July 2021].

Ariel, P. D. (2001) Axisymmetric flow of a second grade fluid past a stretching sheet. *International Journal of Engineering Science*. 39 (5), 529–553. Available from: https://doi.org/10.1016/S0020-7225(00)00052-5.

Bhattacharyya, K. (2012) Mass transfer on a continuous flat plate moving in parallel or reversely to a free stream in the presence of a chemical reaction. *International Journal of Heat & Mass Transfer*. 55 (13-14), 3482–3487. Available from: https:// doi.org /10.1016/j.ijheatmasstransfer.2012.03.005.

Bhattacharyya, K. & Layek, G. C. (2014) Magnetohydrodynamic boundary layer flow of nanofluid over an exponentially stretching permeable sheet. *Physics Research International*. 14 (12), 1–13. Available from: http://dx.doi.org/10.1155/2014/592536.

Bidin, B. & Nazar, R. (2009) Numerical solution of the boundary layer flow over an exponentially stretching sheet with thermal radiation. *European Journal of Scientific Research*. 33 (4), 710–717. Available from: http://www.eurojournals.com/ejsr.html [Accessed 20th April 2022].

Blasius, H. (1950) The boundary layers in fluids with little friction. *Zeitschrift*

Für Mathematik und Physik. 56 (1). Available from: https://ntrs.nasa.gov/api /citations/20050028493/downloads/ 20050028493.pdf. [Accessed 22th July 2021].

Buongiorno, J. (2006) Convective transport in nanofluids. *Journal of Heat Transfer*. 128 (3), 240–250. Available from: https: //doi.org/10.1115 /1.2150834.

Char, M. I. (1988) Heat transfer of a continuous, stretching surface with suction or blowing. *Journal of Mathematical Analysis and Applications*. 135 (2), 568–580. Available from: https://doi.org/10.1016/0022-247X(88)90172-2.

Chen, L. & Xie, H. (2009) Silicon oil based multiwalled carbon nanotubes nanofluid with optimized thermal conductivity enhancement. *Colloids and Surfaces: A Physicochemical and Engineering Aspects*. 352 (1–3), 136–140. Available from: https://doi.org/ 10.1016/ j.colsurfa.2009.10.015.

Choi, S. U. (1995) Enhancing Thermal Conductivity of Fluids with Nanoparticles, In: Siginer, D. A. & Wang, H. P. (eds.) *Developments and Applications of Non-Newtonian Flows. ASME, Vol. 66, 1995*. New York. pp. 99–105. Available from: https://www.scirp.org/ reference/ referencespapers.aspx?referenceid=1073237. [Accessed 28th April 2021].

Crane, L. J. (1970) Flow past a stretching plate. *Zeitschrift für angewandte Mathematik und Physik ZAMP*. 21 (4), 645–647. Available from: https://doi.org/10.1007/BF01587695.

Das, S. K., Choi, S. U. & Patel, H. E. (2006) Heat transfer in nanofluids—a review. *Heat Transfer Engineering*. 27 (10), 3–19. Available from: https://doi.org/10.1080/01457630600904593.

Das, S. K., Putra, N., Thiesen, P. & Roetzel, W. (2003) Temperature dependence of thermal conductivity enhancement for nanofluids. *Jouurnal of Heat Transfer*. 125 (4), 567–574. Available from: https://doi.org/10.1115/1.1571080.

Durst, F. (2008) Introduction, Importance and Development of Fluid Mechanics. In: Springer, Berlin, Heidelberg (ed.) *Fluid Mechanics*. Springer Science and Business Media, 719, pp. 1–14. Available from: https://doi.org/10.1007/978-3-540-71343-2_1.

Elbashbeshy, E. M. A. (2001) Heat transfer over an exponentially stretching continuous surface with suction. *Archives of Mechanics*. 53 (6), 643–651. Available from: https: //am.ippt.pan.pl /index.php /am /article /view/v53p643. [Accessed 29th August 2021].

Falkner, V.M. & Skan, S.W. (1931) Some Approximate Solutions of the Boundary Layer Equations. *Philosophical Magazine*. 12, 865–896. Available from:

https://doi.org/10.1080/14786443109461870.

Gbadeyan J. A., Olanrewaju M. A. & Olanrewaju P. O. (2011) Boundary layer flow of a nanofluid past a stretching sheet with a convective boundary condition in the presence of magnetic field and thermal radiation. *Australian Journal of Basic and Applied Science*. 5 (9), 1323–1334. Available from: https://www.sciencedirect.com/science/article /abs/pii/S1290072911000603. [Accessed 3rd August 2021].

Gupta, P. S. & Gupta, A. S. (1977) Heat and mass transfer on a stretching sheet with suction or blowing. *The Canadian Journal of Chemical Engineering*. 55 (6), 744–746. Available from: https://doi.org/10.1002/cjce.5450550619.

Hassanien, I. A., Abdullah, A. A. & Gorla, R. S. R. (1998) Flow and heat transfer in a power-law fluid over a nonisothermal stretching sheet. *Mathematical and Computer Modelling*. 28 (9), 105–116. Available from: https://doi.org/10.1016/S0895– 7177(98) 001484.

Hayat, T., Abbas, Z. & Sajid, M. (2007) On the Analytic Solution of Magnetohydrodynamic Flow of a Second Grade Fluid Over a Shrinking Sheet. *International Journal of Applied Mechanics*. Available from: https:// asmedigitalcollection.asme.org/appliedmechanics74/6/1165/476041/On the Analytic Solution of Magnetohydrodynamic. [Accessed 22nd August 2021].

Ibrahim, W. (2017) Magneto hydrodynamic (MHD) boundary layer stagnation point flow and heat transfer of a Nano fluid past a stretching sheet with melting. *Propulsion and Power Research* . 6, 214–222. Available from: https://doi.org/10.1016/j.jppr.2017.07.002.

Ishak, A., Nazar, R. & Pop, I. (2007) *Boundary layer on a moving wall with suction and injection*. Chinese Physics Letters, 24 (8), 2274. Available from: https://iopscience.iop.org /article /10.1088 /0256-307X /24/8/033/meta. [Accessed 30thAugust 2021].

Kakac, S. & Pramuanjaroenkij, A. (2009) Review of convective heat transfer enhancement with nanofluids. *International Journal of Heat and Mass Transfer*. 52 (13–14), 3187–3196. Available from: https://doi.org /10.1016/ j.ijheatmasstransfer.2009.02.006.

Keller, H. B. (1978) Numerical methods in boundary-layer theory. *Annual Review of Fluid Mechanics*. 10 (1), 417–433. Available from: https://doi.org/10.1146/ annurev.fl.10.010178.002221.

Khan, W. A. & Pop, I. (2011) Flow and heat transfer over a continuously moving flat plate in a porous medium. *Journal of Heat Transfer*. 133 (5). Available from:

https://doi.org/10.1115/1.4003012.

Kundu, P. K. & Cohen, I. M. (2000) *Fluid Mechanics*. Elsevier Axad, Press London, 18 (2000), 199–400. Available from: http://ftp.demec.ufpr.br/disciplinas/EMEC7033/Fluid Mechanics KUNDU pdf. [Accessed 30th March 2022].

Mahapatra, T. R. & Gupta, A. S. (2002) Heat transfer in stagnation-point flow towards a stretching sheet.*Heat and Mass Transfer*, 38, 517–521. Available from: https://doi.org/ 10.1007/ s002310100215.

Masuda, H., Ebata, A., Teramae, K. & Hishinuma, N. (1993) Alteration Of Thermal Conductivity And Viscosity Of Liquid By Dispersing Ultra-Fine Particles. Dispersion of Al_2O_3, SiO_2 and TiO_2 Ultra-Fine Particles. *Journal of Physics Letters*. 7, 227–233. Available from: https://www.sid.ir/en/Journal/ViewPaper.aspx?ID=312989. [Accessed 2nd August 2021].

Matta, A. & Gajjela, N. (2018) Order of chemical reaction and convective boundary condition effects on micropolar fluid flow over a stretching sheet. *AIP Advances*, 8 (11), 115212. Available from: https://doi.org/10.1063/1.5053445.

Maxwell, J. C. (1904) *A Treatise on Electricity and Magnetism*. Oxford University Press Cambridge, 1, 435–441. Available from: https://www.aproged.pt/biblioteca/ MaxwellI.pdf. [Accessed 30th January 2022].

Muhammad, K. A. M., Noar, N. A. Z. M., Salleh, M. Z., & Ishak, A. (2014) Boundary Layer flow over a moving plate in a Nano fluid with viscous dissipation. *The 3rd International Conference on Computer Engineering and Mathematical Sciences*. pp. 883–889. Available from: https://doi.org/10.1063/1.5053445.

Mukhopadhyay, S. (2012) Mixed convection boundary layer flow along a stretching cylinder in porous medium. *Journal of Petroleum Science and Engineering*. 96, 73–78. Available from: https://www.sciencedirect.com/science/article/abs/pii/S0920410512001945. [Accessed 30th October 2021].

Namburu, P. K., Kulkarni, D. P., Misra, D. & Das, D. K. (2007) Viscosity of copper oxide nanoparticles dispersed in ethylene glycol and water mixture. *Experimental Thermal and Fluid Science*. 32 (2), 397–402. Available from: https://doi.org /10.1016/ j.expthermflusci.2007.05.001.

Nguyen, C .T. , Desgranges, F., Galanis, N., Roy, G., Maré, T., Boucher, S. & Mintsa, H. A.

(2008) Viscosity data for Al_2O_3 water nanofluid—hysteresis: is heat transfer enhancement using nanofluids reliable? *International Journal of Thermal Sciences*. 47(2), 103–111. Available from: https://doi.org/10.1016/j.ijthermalsci.2007.01.033.

Pavlov, K. B. (1974) Magnetohydrodynamic flow of an incompressible viscous fluid caused by deformation of a plane surface. Magnitnaya Gidrodinamika, *Journal of Applied Mathematics and Mechanics*. 4 (1), 146–147. Available from: http://doi.org/10.22364 /mhd.

Pal, D. (2009) Heat and mass transfer in stagnation-point flow towards a stretching surface in the presence of buoyancy force and thermal radiation. *Meccanica*. 44(2), 145–158. Available from: https:// link.springer.com /article /10.1007/s11012-008-9155-1. [Accessed 30th November 2021].

Prandtl, L. (1904) *Verhandlungen des dritten Internationalen Mathematiker-Kongresses*. Heidelberg, Leipeizig, 484–491. Available from: https://www.scirp.org /(S(vtj3fa45qm1ean45vvffcz55))/reference/ReferencesPapers.aspx?ReferenceID=1642454. [Accessed 30th April 2022].

Renn, J., Damerow, P. & McLaughlin, P. (2002) February. Aristotle, Archimedes, Euclid, and the origin of mechanics: The perspective of historical epistemology. In: José Luis (ed.) *Symposium Arqumedes*. Montesinos Sirera, pp. 43–59. Available from: https://citeseerx.ist.psu.edu/viewdoc/doi/10.1.1.580.8174 pdf. [Accessed 22nd January 2022].

Rosenhead, L. (1963) *Laminar boundary layers: an account of the development, structure, and stability of laminar boundary layers in incompressible fluids, together with a description of the associated experimental techniques*. Clarendon Press. Available from: https://www.worldcat.org/title/laminar boundary layer/oclc/1471689. [Accessed 30th December 2021].

Ryzhov, O. S. & Savenkov, I. V. (1987) *Asymptotic theory of a wave packet in a boundary layer on a plate*. *Journal of Applied Mathematics and Mechanics* . 51(5), 644–651. Available from: https://doi.org/10.1016/ 0021-8928(87)900177.

Sakiadis, B. C. (1961) Boundary layer behavior on continuous solid surfaces: boundary layer equations for two dimensional and axisymmetric flows. *AIChE Journal*. 7, 26–28. Available from: https://www.scirp.org/(S(i43dyn45teexjx455qlt3d2q))/reference /ReferencesPapers.aspx?ReferenceID=1532727. [Accessed 22nd Feburary 2022].

Schlichting, H. & Gersten, K. (2003) *Boundary Layer Theory*. New York:

McGraw-Hill, 573.Available from: http://ae.sharif.edu/viscousflow/Schlichting Boundary Layer Theory.pdf. [Accessed 22nd March 2022].

Schowalter, W. R. (1960) The application of boundary-layer theory to power-law pseudoplastic fluids: Similar solutions. *AIChE Journal*. 6 (1), 24–28. Available from: https://doi.org/10.1002/aic.690060105.

Sheremet, M. A. & Pop, I. (2014) Conjugate natural convection in a square porous cavity filled by a nanofluid using Buongiorno's mathematical model. *International Journal of Heat and Mass Transfer*. 79, 137–145. Available from: https://doi.org/ 10.1016 /j.ijheatmasstransfer.2014.07.092.

Sundar, L. S., Singh, M. K. & Sousa, A. C. (2013) Investigation of thermal conductivity and viscosity of Fe_3O_4 nanofluid for heat transfer applications. *International Communications in Heat and Mass Transfer*. 44, 7–14. Available from: https://doi.org/10.1016/ j.icheatmasstransfer. 2013.02.014.

Tani , I. (1977) History of boundary layer theory. *Annual review of fluid mechanics*. 9 (1), 87–111. Available from: https://doi.org/10.1146/annurev.fl.09.010177.000511.

Timoshin, S. N. (1997) Instabilities in a high-Reynolds-number boundary layer on a film-coated surface. *Journal of Fluid Mechanics*. 353, 163–195. Available from: https://doi.org/10.1017/ S0022112097007337.

Vafai, K. & Hadim, H. (2000) Overview of current computational studies of heat transfer in porous media and their applications natural convection and mixed convection. *Journal of Advanced Numerical Heat Transfer*. 2, 331–371. Available from: https://www.taylorfrancis.com/chapters/edit/10.1201/9781315136882-10. [Accessed 15th November 2021].

Vajravelu, K. & Mukhopadhyay, S. (2015) *Fluid flow, heat and mass transfer at bodies of different shapes: Numerical solutions*. Academic Press, pp. 185. Available from: https://www.elsevier.com/books/fluid flow heat and mass transfer at bodies of different shapes/vajravelu/978-0-12-803733-1. [Accessed 30th November 2021].

Vajravelu, K., Prasad, K. V. & Ng, C. O. (2012) Unsteady flow and heat transfer in a thin film of Ostwald–de Waele liquid over a stretching surface. *Communications in Nonlinear Science and Numerical Simulation*. 17 (11), 4163–4173. Available from: https://doi.org/10.1016/j.cnsns.2012.01.027.

Wang, L. (2009) A further study on the non-linear dynamics of simply supported pipes

conveying pulsating fluid. *International Journal of Non-Linear Mechanics*, 44 (1), 115–121. Available from: https://doi.org/10.1016/j.ijnonlinmec.2008.08.010.

Xie, H. Q., Wang, J. C., Xi, T. G. & Liu, Y. (2002) Thermal conductivity of suspensions containing nanosized SiC particles. *International Journal of Thermophysics*. 23 (2), 571–580. Available from: https://aip.scitation.org/doi/abs/10.1063/1.1454184. [Accessed 20th November 2021].

Yu, W., Xie, H., Chen, L. & Li, Y. (2009) Investigation of thermal conductivity and viscosity of ethylene glycol based ZnO nanofluid. *Thermochimica Acta*. 491 (1–2), 92–96. Available from: https://doi.org/10.1016/j.tca.2009.03.007.

Zheng, L., Niu, J., Zhang, X. & Ma, L. (2012) Dual solutions for flow and radiative heat transfer of a micropolar fluid over stretching/shrinking sheet. *International Journal of Heat and Mass transfer*. 55 (25–26), 7577–7586. Available from: https://doi.org/10.1016/j.ijheatmasstransfer.2012.07.067.

CERTIFICATE
(SYNOPSIS APPROVAL CERTIFICATE)

This is to certify that the Synopsis for Ph.D. Thesis titled, "COMPUTATIONAL STUDY OF BOUNDARY LAYER FLOW OF NANOFLUIDS OVER STRETCHING SURFACES", submitted by Ms. Rekha Devi (H14123) to the Career Point University, Hamirpur (H.P.), India in partial fulfillment of the requirements for the degree of DOCTOR OF PHILOSOPHY in the Department of Mathematics &Statistics, School of Basic & Applied Sciences has been approved by the Research Advisory Committee (RAC) after an Oral Examination of the student on dated 2 May, 2022.

Dr. Shilpa Sood
Associate Professor
Department of Mathematics

(Supervisor)

Research Advisory Committee

Dr. Rahul Sharma
Assistant Professor
Department ofChemistry

(Member)

Dr. Indu Sharma
Associate Professor
Department of Physics

(Member)

Dr. Shilpa Sood
Associate Professor
Department of Mathematics

(Head)

Dr. Kuldeep Kumar
Associate Professor
Department ofChemistry

(Dean/Assoc Dean R & D)